Decay of the Loschmidt echo in a time-dependent environment

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We study the decay rate of the Loschmidt echo or fidelity in a chaotic system under a time-dependent perturbation $V(\mathbf{q},t)$ with typical strength $\hbar/\tau_V$. The perturbation represents the action of an uncontrolled environment interacting with the system, and is characterized by a correlation length ξ_0 and a correlation time τ_0. For small perturbation strengths or rapid fluctuating perturbations, the Loschmidt echo decays exponentially with a rate predicted by the Fermi "golden rule," $1/\tilde{\tau}=\tau_c/\tau_V^2$, where $\tau_c \sim \min[\tau_0, \xi_0/v]$ and v is the typical particle velocity. Whenever the rate $1/\tilde{\tau}$ is larger than the Lyapunov exponent of the system, a perturbation independent Lyapunov decay regime arises. We also find that by speeding up the fluctuations (while keeping the perturbation strength fixed) the fidelity decay becomes slower, and hence one can protect the system against decoherence.

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I. INTRODUCTION

The time evolution of a quantum system is quite robust to perturbations of the system initial conditions, regardless if its underlying dynamics is integrable or chaotic [1]. This is in deep contrast to classical evolution, particularly that of a chaotic system. In a seminal paper, Peres [2] noticed that the quantum time evolution is also sensitive to differences between chaotic and integrable dynamics, but in a peculiar setup: One needs to examine the overlap of identically prepared states evolving under slightly different Hamiltonians. The modulus square of this overlap, called Loschmidt echo (LE) or fidelity [3], measures the recovery obtained when a wave packet evolves for a time t, followed by a backwards evolution with a perturbed Hamiltonian for the same time interval.

A considerable number of investigations have been devoted to study the phenomena related to the LE, in particular the different regimes that arise depending on the perturbation strength. For very weak perturbations, the LE is described by standard perturbation schemes and a Gaussian decay is observed. For stronger perturbations, where perturbation theory breaks down, large phase fluctuations [3] lead to an exponential decay of the LE described by the Fermi "golden rule" (FGR) [4,5]. For even stronger perturbations, but still weak in the classical sense, a semiclassical analysis yields an exponential LE decay that does not depend on the perturbation strength: The decay rate is determined by the Lyapunov exponent that characterizes the classical counterpart of the unperturbed system [3]. The latter two cases are called the FGR and Lyapunov regimes, respectively. The LE decay rate is the minimum between the width of the local density of states (LDOS), as given by the FGR and the Lyapunov exponent [3–5]. These findings were verified numerically in a number of systems [4–8]. The theory is successful to the extend that, by analyzing the LE decay, the quantum evolution of a system can be used to quantitatively assess its classical Lyapunov exponent [5,9].

The theory was later extended to classically integrable systems [10], for which a power-law-like decay is predicted. This result is still somewhat controversial [11], since integrable systems, as a rule, display nongeneric features [12]. In any event, there are several indications that the LE decay is very different whether the underlying classical system has a chaotic [4–8], integrable [10–12], or mixed phase space [13].

Albeit this wealth of interesting results, so far the theory of the LE nonperturbative regime has only dealt with time-independent perturbations. The most probable motivation for this restrictive choice can be traced back to the experiments that triggered the research on the LE problem [14,15]: They studied the time reversal of many-spin dynamics, where the perturbation is simply a static part of the Hamiltonian.

Numerous physical situations call for an extension of the LE theory that accounts for a time-dependent perturbation. Let us mention a few. Experimentally, a subsystem selected from a large spin system with many-body interactions can be represented as immersed in an external fluctuating potential [16]—the same approximation holds whenever the uncontrolled degrees of freedom are those of an environment with complex dynamics [17]. Formally, the current analytical description contrasts with numerical results [18] observed in periodically kicked one-dimensional models [4,8], where the perturbation can be interpreted as time dependent. The LE decay due to a time-dependent environment has also connections to the problem of decoherence in open systems [19–21], in mesoscopic physics [22], and possibly in quantum computation [23–25]. Indeed, the decay of the LE is related to the decay of quantum correlations by an external environment and the quantum-classical correspondence, as can be shown using the Wigner function representation [20,26–28].

In this work we use the semiclassical approximation to derive the LE decay in the presence of a time-dependent perturbation, generalizing the approach presented in Ref. [3]. We show that the existence of a LE perturbation-independent regime is quite generic. For that purpose, instead of using a particular model, we use a statistical approach. We obtain a closed expression for the LE decay in the FGR regime using simple assumptions about the perturbation autocorrelation function. We conclude by discussing the different limits of our results and the seemingly strange feature that faster fluctuations of the perturbation or stronger chaos in the system lead to a slower decay of the Loschmidt echo.

II. LOSCHMIDT ECHO IN A TIME-DEPENDENT ENVIRONMENT

The object of interest, the Loschmidt echo, is defined as

$$M(t) = |\langle \psi_0 | U(t_0,t) U_0(t,t_0) | \psi_0 \rangle|^2, \tag{1}$$

where $|\psi_0\rangle$ is an arbitrary wave packet prepared at time t_0. For simplicity, and in line with Ref. [3], we choose the initial state $|\psi_0\rangle$ as a Gaussian wave packet centered at an arbitrary point $\mathbf{r}_0$ with dispersion σ and initial momentum $\mathbf{p}_0$. Numerical studies consider other kinds of localized states in phase space [29,30], evolved states [10], and even eigenstates of H_0 [4,27] and obtain results consistent with Ref. [3]. This observation can, in principle, be justified by using the dephasing representation [31] which deals with any kind of initial state. In Eq. (1), U_0 is the standard time evolution operator, namely,

$$U_0(t,t_0) = T \exp\left(-\frac{i}{\hbar} \int_{t_0}^{t} dt' H_0(t') \right), \tag{2}$$

where T is the time ordering operator, while

$$U(t_0,t) = \bar{T} \exp\left(-\frac{i}{\hbar} \int_{t}^{t_0} dt' H(t') \right), \tag{3}$$

with $\bar{T}$ the inverse time ordering operator. Equation (1) is often viewed as the fidelity of two wave packets prepared at the same initial state and evolving forward in time under different Hamilton operators.

In general, time ordering makes the exact evaluation of $M(t)$ for a time-dependent Hamiltonian a daunting task. To circumvent this difficulty we employ the semiclassical approximation, in which time ordering is trivially accounted for by taking the time evolution of classical trajectories, as we detail in the sequel.

We consider the Hamiltonian H defined as

$$H = H_0 + V(\mathbf{q},t), \tag{4}$$

where H_0 is a time-independent Hamiltonian that displays chaotic motion in the classical limit and $V(\mathbf{q},t)$ is the time-dependent perturbation potential or the system interaction with a complex environment. We note that the time scales that characterize the Loschmidt echo decay (the time scale on which decoherence occurs) are assumed to be much shorter than any relaxation-to-the-environment time scale. Thus heating effects due to the driving of the perturbation [32] are neglected.

The semiclassical propagator reads [33]

$$\langle \mathbf{q}' | U(t) | \mathbf{q} \rangle = \left(\frac{1}{2\pi\hbar i} \right)^{d/2} \sum_{s(\mathbf{q}',\mathbf{q},t)} C_s^{1/2} \times \exp\left(\frac{i}{\hbar} S_s(\mathbf{q}',\mathbf{q},t) - \frac{i\pi}{2}\alpha_s \right), \tag{5}$$

where s is a classical path that spends a time t to travel from $\mathbf{q}$ to $\mathbf{q}'$, S_s is the action (Hamilton principal function) along trajectory s, given by $S_s(\mathbf{q}',\mathbf{q},t) = \int_0^t d\tau L(\dot{\mathbf{q}}_s(\tau),\mathbf{q}_s(\tau),\tau)$, α_s is the number of conjugate points along s, and $C_s = |\det \frac{\partial \mathbf{p}_s}{\partial \mathbf{q}'_s}|$ is the Jacobian of the phase-space transformation between the initial momentum $\mathbf{p}_s(0)$ and the final position $\mathbf{q}'_s(t)$—a density of classical paths [33].

It is only possible to proceed analytically if we restrict ourselves to the regime of weak perturbations. Let us approximate the action along a given trajectory s by

$$S_s(t) \simeq S_s^0(t) - \int_0^t dt' V(\mathbf{q}_s(t'),t'), \tag{6}$$

where $S_s^0(t)$ refers to the action corresponding to s obtained from H_0 and $\mathbf{q}_s(t)$ gives the particle position along the unperturbed trajectory s as a function of time. Equation (6) results from a classical perturbation theory analysis and gives the lowest order correction to the action in "powers" of V. It can be derived in different ways, like for instance, by employing a phase-space variational principle [34]. Corrections $\delta S_s(t)$ accounting for the bending of s due to V are of higher order in the action $S_s(t)$. Albeit parametrically small in V, for sufficiently large times we expect such terms to become significant. Unfortunately, for chaotic systems a reliable estimate of the typical magnitude of $\delta S_s(t)$ is difficult. Note, however, that since the actions in Eq. (5) are measured in units of $\hbar$, even a very weak classical perturbation can cause large quantum variations. In addition, as we will see, the time interval of interest ranges from the Ehrenfest time (so that many orbits contribute) up to a time of the order of the inverse Lyapunov exponent (that is classically small). Hence it is quite plausible to find a comfortable parameter range for V where Eq. (6) holds. Indeed, numerical investigations show that the classical perturbation scheme works surprisingly well [5,35] to the extent that no significant discrepancies were ever observed [37]. These numerical findings have been related to the structural stability of the manifold of trajectories in phase space [38]: Even though individual trajectories are exponentially sensitive to perturbations, one can always find a "replacement" trajectory in the manifold that joins the points of interest for a given time interval [29].

Our calculation proceeds along the lines of Ref. [3], which we now briefly sketch. We assume that the wave packet $\langle \mathbf{r} | \psi_0 \rangle$ is well localized, $\xi \gg \sigma \gg \lambda_B$, where ξ is a typical length of the perturbation (in Ref. [3] the width of Gaussian impurities) and λ_B is the de Broglie wavelength of the particle. Neglecting terms with a rapidly oscillating phase, one arrives at the semiclassical expression for the Loschmidt echo,

$$M(t) \equiv |O(t)|^2 \simeq \left(\frac{\sigma^2}{\pi\hbar^2} \right)^d \left| \int d\mathbf{r} \sum_{s(\mathbf{r},\mathbf{r}_0,t)} C_s \exp\left[\frac{i}{\hbar} \Delta S_s(t) \right] \times \exp\left[-\frac{\sigma^2}{\hbar^2} (\bar{\mathbf{p}}_s - \mathbf{p}_0)^2 \right] \right|^2, \tag{7}$$

where ΔS_s is the action difference between trajectories evolved with H_0 and H, and $\mathbf{p}_s = -\partial S_s^0 / \partial \mathbf{r}_0$. All trajectories s

start at $\mathbf{r}_0$, the position where the Gaussian wave packet $\langle \mathbf{r}|\psi_0\rangle$ is centered at. For short times $M(t)$ can only capture the local instabilities of the classical dynamics, thus, it shows large fluctuations [5]. By averaging over an ensemble of perturbations, one obtains $\langle M(t)\rangle$ that puts in evidence the exponential decay [26].

The Lyapunov and the FGR decay regimes are related to the different ways of pairing the path summations in the double sum of Eq. (7). This will become evident from the analysis of $\langle M(t)\rangle$ written as

$$\langle M(t)\rangle = \langle M^{nd}(t)\rangle + \langle M^{d}(t)\rangle, \tag{8}$$

and define $\langle M^{nd}(t)\rangle \equiv |\langle O(t)\rangle|^2$ and $\langle M^{d}(t)\rangle \equiv \langle |O(t)|^2\rangle - |\langle O(t)\rangle|^2$. The first term, $\langle M^{nd}(t)\rangle$, contains an average of the overlap $O(t)$ and does not account for action correlations between the trajectories s and s' in the double sum of Eq. (7). The second term, $\langle M^{d}(t)\rangle$ corrects for correlations between trajectories, and is more subtle to compute. Reference [3] offers a simple interpretation to both terms: $\langle M^{nd}(t)\rangle$ is dominated by trajectories where s and s' lie far apart in phase space (nondiagonal in trajectories) so that the action differences can be considered uncorrelated, whereas $\langle M^{d}(t)\rangle$ stems from terms where the action differences ΔS_s and $\Delta S_{s'}$ are correlated (diagonal terms).

A. Nondiagonal contributions to $\langle M(t)\rangle$

Let us first calculate $\langle M^{nd}(t)\rangle = |\langle O(t)\rangle|^2$, that reads

$$\langle M^{nd}(t)\rangle \simeq \left(\frac{\sigma^2}{\pi\hbar^2}\right)^d \left| \int d\mathbf{r} \sum_{s(\mathbf{r},\mathbf{r}_0,t)} C_s \left\langle \exp\left[\frac{i}{\hbar}\Delta S_s(t)\right]\right\rangle \times \exp\left[-\frac{\sigma^2}{\hbar^2}(\bar{\mathbf{p}}_s - \mathbf{p}_0)^2\right]\right|^2, \tag{9}$$

where $\langle\cdots\rangle$ indicates that we average over an ensemble of perturbations.

We assume, as is customary for chaotic systems, that the actions for different paths are Gaussian distributed [29,39]. This leads to an enormous simplification, allowing us to write

$$\left\langle \exp\left[\frac{i}{\hbar}\Delta S_s(t)\right]\right\rangle \simeq \exp\left[-\frac{1}{2\hbar^2}\langle[\Delta S_s(t)]^2\rangle\right]. \tag{10}$$

We remain with the task of evaluating the action variance

$$\langle[\Delta S_s(t)]^2\rangle = \int_0^t dt' \int_0^t dt'' \langle V(\mathbf{q}_s(t'),t')V(\mathbf{q}_s(t''),t'')\rangle. \tag{11}$$

For that purpose we introduce an ensemble of perturbations V to model the general features of the environment.

In order to keep our calculation as general as possible, we assume very little knowledge of the perturbation, requiring only that time and space correlations are independent, viz.

$$\langle V(\mathbf{q},t)V(\mathbf{q}',t')\rangle = \langle V^2\rangle C_S(|\mathbf{q}-\mathbf{q}'|)C_T(|t-t'|). \tag{12}$$

The typical perturbation strength is $\langle V^2\rangle^{1/2}$, and $\tau_V = \hbar/\langle V^2\rangle^{1/2}$ is its associated time scale. The dimensionless functions C_S and C_T quantify the spatial and time correlations of the potential $V(\mathbf{q},t)$. We further require that C_S or C_T decay sufficiently fast, so that

$$\int_0^\infty dr\, r^{d-1} C_S(r) < \infty \text{ and } \int_0^\infty dt C_T(t) < \infty, \tag{13}$$

where d is the system dimension [36].

To guide the discussion, let us introduce the correlation length ξ_0 and the correlation time τ_0 that characterize C_S and C_T, respectively. To compare time and length scales we also introduce v, the typical particle velocity in the system. To simplify the discussion, we only consider here the case when the decay of correlations is induced by the fluctuations of V. However, we note that conditions (13) can also be justified by the chaotic dynamics of H_0 [4,6–8,26]. Hence our results are valid not only for random perturbations, but also for static and periodic ones.

In the limit of $\tau_0 \gg 1/\lambda$ the perturbation is quasistatic and the results of Ref. [3] hold without further change. We are interested in the regime where the typical times of the perturbation are comparable to those of the system, $\tau_0 \lesssim 1/\lambda$.

After the ensemble average, Eq. (11) becomes

$$\langle \Delta S_s(t)^2\rangle = \langle V^2\rangle \int_0^t d\bar{t} \int_{-\infty}^{\infty} d\tau C_S[|\mathbf{q}_s(\bar{t}-\tau/2) - \mathbf{q}_s(\bar{t}+\tau/2)|]C_T(\tau), \tag{14}$$

where we considered times t much larger than τ_0 and ξ_0/v, which allows us to take the integral in τ from $-\infty$ to $+\infty$. Equation (14) has two limiting regimes that are readily solved. In the first one, the spatial disorder has a much shorter scale than the temporal one: $\tau_0 \gg \xi_0/v = \tau_\xi$. In this case we re-obtain the result of Ref. [3],

$$\langle \Delta S_s(t)^2\rangle \simeq \langle V^2\rangle \int_0^t d\bar{t} \int_{-\infty}^{\infty} d\tau C_S[|\mathbf{q}_s(\bar{t}-\tau/2) - \mathbf{q}_s(\bar{t}+\tau/2)|] = \frac{t}{\tilde{\tau}_1}\hbar^2, \tag{15}$$

where $C_T(\tau)$ is assumed constant, and $\tilde{\tau}_1$ is defined as

$$\frac{1}{\tilde{\tau}_1} = \frac{\tau_\xi}{\tau_V^2}. \tag{16}$$

When $\tau_0 \ll \tau_\xi$, we deal with the opposite regime, and

$$\langle \Delta S_s(t)^2\rangle \simeq \langle V^2\rangle \int_0^t d\bar{t} \int_{-\infty}^{\infty} d\tau C_T(\tau) = \frac{t}{\tilde{\tau}_2}\hbar^2, \tag{17}$$

with

$$\frac{1}{\tilde{\tau}_2} = \frac{\tau_0}{\tau_V^2}. \tag{18}$$

The calculation of $\langle M^{nd}(t)\rangle$ is now straightforward: We insert Eq. (15) or Eq. (17) into Eq. (9), use the Jacobian C_s to change variables, and perform a simple Gaussian integral to obtain

$$\langle M^{nd}(t)\rangle = \exp\left(-\frac{t}{\tilde{\tau}_i}\right), \qquad (19)$$

where $i=1$ if $\tau_0 \gg \tau_\xi$ or $i=2$ if $\tau_0 \ll \tau_\xi$. The exponential decay given by Eq. (19) is often called Fermi golden rule decay [4,5]. In the situations discussed above, the FGR exponent changes from being governed by the spatial to the temporal correlations of $V(\mathbf{q},t)$. The interesting "correlation crossover regime"—where neither the temporal nor the spatial correlation dominate—will be discussed shortly for a particular form of C_S and C_T.

B. Diagonal contributions to $\langle M(t)\rangle$

The average diagonal (in trajectories) contribution to $M(t)$ is

$$\langle M^d(t)\rangle = \langle|O(t)|^2\rangle - |\langle O(t)\rangle|^2 \simeq \left(\frac{\sigma^2}{\pi\hbar^2}\right)^d \int d\mathbf{r}\int d\mathbf{r}' \sum_{\substack{s(\mathbf{r},\mathbf{r}_0,t)\\ s'(\mathbf{r}',\mathbf{r}_0,t)}} C_s \times C_{s'}\left\langle \exp\left[\frac{i}{\hbar}[\Delta S_s(t)-\Delta S_{s'}(t)]\right]\right\rangle_c \times\exp\left[-\frac{\sigma^2}{\hbar^2}[(\bar{\mathbf{p}}_s-\mathbf{p}_0)^2+(\bar{\mathbf{p}}_{s'}-\mathbf{p}_0)^2]\right]. \qquad (20)$$

As before, $\langle\cdots\rangle$ stands for the average over an ensemble of perturbations V, whereas $\langle AB\rangle_c \equiv \langle AB\rangle - \langle A\rangle\langle B\rangle$. The latter picks only the correlations (or the connected parts) between A and B. As a consequence, in Eq. (20) the sums over trajectories s and s' are effectively constrained and run only over the pairs s and s' that are correlated, i.e., that remain close in phase space.

Let us calculate the average $\langle\cdots\rangle_c$ appearing in Eq. (20). As in Eq. (10) we use the standard assumption that

$$\left\langle \exp\left\{\frac{i}{\hbar}[\Delta S_s(t)-\Delta S_{s'}(t)]\right\}\right\rangle_c \simeq \exp\left\{-\frac{1}{2\hbar^2}\langle[\Delta S_s(t)-\Delta S_{s'}(t)]^2\rangle\right\}. \qquad (21)$$

We also use Eq. (6) to write

$$\Delta S_s(t)-\Delta S_{s'}(t) = \int_0^t dt'[V(\mathbf{q}_s(t'),t')-V(\mathbf{q}_{s'}(t'),t')]. \qquad (22)$$

As the two trajectories s and s' to remain close, we can expand $V(\mathbf{q}_s(t),t)$ to first order around s and obtain

$$\Delta S_s(t)-\Delta S_{s'}(t) \simeq \int_0^t dt'\, \boldsymbol{\nabla} V(\mathbf{q}_s(t'),t')\cdot[\mathbf{q}_s(t')-\mathbf{q}_{s'}(t')]. \qquad (23)$$

To calculate the action difference variance we turn our attention to the force correlation function, namely,

$$C_{\boldsymbol{\nabla}}(|\mathbf{q}-\mathbf{q}'|,|t-t'|) \equiv \langle \boldsymbol{\nabla} V[\mathbf{q},t]\cdot\boldsymbol{\nabla} V[\mathbf{q}',t']\rangle. \qquad (24)$$

As before, we introduce an ensemble of perturbations, and write

$$C_{\boldsymbol{\nabla}}(|\mathbf{q}-\mathbf{q}'|,|t-t'|) = \langle V^2\rangle C_T(|t-t'|)(\boldsymbol{\nabla}_{\mathbf{q}}\cdot\boldsymbol{\nabla}_{\mathbf{q}'})C_S(|\mathbf{q}-\mathbf{q}'|), \qquad (25)$$

such that $(\boldsymbol{\nabla}_{\mathbf{q}}\cdot\boldsymbol{\nabla}_{\mathbf{q}'})C_S(|\mathbf{q}-\mathbf{q}'|)$ decays sufficiently fast, in the sense defined by Eq. (13).

As time evolves, the separation between the coordinates $\mathbf{q}_s(t)$ and $\mathbf{q}_{s'}(t)$ grows as $e^{\lambda t}$, where λ is the largest Lyapunov exponent of H_0. As a result, after some algebra,

$$\left\langle \exp\left[\frac{i}{\hbar}[\Delta S_s(t)-\Delta S_{s'}(t)]\right]\right\rangle \simeq \exp[-A(\mathbf{r}-\mathbf{r}')^2/\hbar^2], \qquad (26)$$

with

$$A = \langle V^2\rangle\tau_0\frac{(1-e^{-2\lambda t})}{2\lambda} \qquad (27)$$

when C_T dominates the decay of $C_{\boldsymbol{\nabla}}$, and

$$A = \langle V^2\rangle\frac{1-e^{-2\lambda t}}{2\lambda v}\int_{-\infty}^{\infty} dq\left[\frac{1-d}{q}\frac{\partial C_S(q)}{\partial q}-\frac{\partial^2 C_S(q)}{\partial q^2}\right], \qquad (28)$$

when C_T decays slowly.

Inserting Eq. (26) in Eq. (20) we can perform the Gaussian integral over $(\mathbf{r}-\mathbf{r}')$, use one of the Jacobians C_s to change variables and replace the other one with a limiting extrapolating form $C_s \sim (\frac{m}{t})^d \exp(-\lambda t)$ [3]. The final step is to compute a Gaussian integral over momenta to obtain

$$\langle M^d(t)\rangle \simeq \bar{A}\exp(-\lambda t), \qquad (29)$$

where $\bar{A}=[m\sigma/(A^{1/2}t)]^d$, and λ is the classical Lyapunov exponent of the system.

In summary, replacing Eqs. (19) and (29) in Eq. (8), we find that the main result of Ref. [3] holds, namely,

$$M(t) = \bar{A}\exp(-\lambda t) + B\exp(-t/\tilde{\tau}), \qquad (30)$$

where $1/\tilde{\tau}$ is given by Eq. (14). The exponential decay of the LE is dominated by the smallest between $1/\tilde{\tau}$ and λ, giving a crossover from FGR to Lyapunov decay as the perturbation strength increases.

C. Correlation crossover

In the regime where $\tau_0 \approx \tau_\xi$, one can only obtain further insight by assuming a specific form of the correlation functions. Although it is a less general result, one can still encompass a broad class of possible perturbations whose correlator decay in a particular way. We will consider the case where both C_S and C_T have Gaussian shapes,

$$\langle V(\mathbf{q},t)V(\mathbf{q}',t')\rangle = \frac{\langle V^2\rangle}{\pi}\exp\left(-\frac{|\mathbf{q}-\mathbf{q}'|^2}{\xi_0^2}\right)\exp\left(-\frac{|t-t'|^2}{\tau_0^2}\right). \tag{31}$$

Under the assumption that t is large compared to τ_0 and τ_ξ, we replace in Eqs. (14) and (21), and obtain the decay rate for the FGR regime,

$$\frac{1}{\tilde{\tau}} = \frac{\tau_V^{-2}}{\sqrt{\tau_0^{-2}+\tau_\xi^{-2}}}, \tag{32}$$

and the prefactor A of the Lyapunov regime:

$$A = \left(\frac{\hbar^2}{v^2\lambda\tilde{\tau}^3}\right)\left(\frac{1-e^{-2\lambda t}}{\sqrt{\pi}}\right)\tau_V^4\left(\frac{d-1}{\tau_\xi^4}+\frac{d}{\tau_0^2\tau_\xi^2}\right). \tag{33}$$

When the temporal or spatial correlation dominate, we recover the previous limit

$$\frac{1}{\tilde{\tau}} \simeq \frac{\tau_c}{\tau_V^2} \text{ with } \tau_c = \min[\tau_0,\tau_\xi]. \tag{34}$$

As before, if the effective time scale τ_c becomes too short, the perturbation cancels itself out causing a very slow decay. This result is consistent with studies of time-dependent errors in a quantum computer [25], where the dynamical decoupling to the environment was interpreted as a manifestation of the quantum Zeno effect [16,40]. Notice that when τ_c is dominated by the dynamics of H_0, the fluctuations become faster for chaotic systems with a larger λ [26].

III. CONCLUSIONS

We have extended the semiclassical theory of the Loschmidt echo to cope with time-dependent perturbations. We expect our results to remain valid in more complex or analytically difficult cases, suitable only for numerical studies. Our treatment is sufficiently general as to describe the situations where the perturbation is the random effect of an uncontrolled environment on the system. The fluctuations we considered could arise either from an explicit time dependence of the perturbation potential, or from the ergodic nature of H_0. In the last case, the underlying chaotic dynamics mimics the randomness required for the decay of the correlation functions. Thus our results should also apply to periodic or very simple oscillating perturbations.

We showed that the Loschmidt echo Lyapunov regime is barely affected by the time dependence of the perturbation, except for prefactors: The decay is dominated by the system's intrinsic dynamics of stretching and folding. In the FGR regime—when the nondiagonal terms dominate—the spatial and time scales of the perturbation compete with each other, and a simple behavior can be extracted when the relevant scales are far apart. In the intermediate regime, where the scales are comparable, using a simple (yet general) example we compute the decay rate of $M(t)$. The form of Eq. (34) stresses how fast fluctuations lead to self-cancellation of the interaction with the environment. In the case of the LE, a vanishing FGR exponent prevents the appearance of the perturbation independent Lyapunov regime. Surprisingly, this happens not only for rapidly fluctuating perturbations, but also by increasing the Lyapunov exponent. The slowing down of the FGR regime of decoherence—induced by fast fluctuations—was recently experimentally measured in NMR experiments [41], where a connection to the quantum Zeno effect was observed. It is interesting to recall that dynamical decoupling to the environment is what makes liquid NMR quantum computers possible (albeit small). The fast random movements of the molecules in the liquid average out the more difficult to control dipolar interactions present, e.g., in solids. Our work points to the importance of exploring dynamical alternatives to suppress quantum decoherence [40,42].

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